for any two functions ψ and ϕ. To prove the above equation, for Hermitian operator A and state function $(\psi+\phi)$, we have,

$$\int(\psi+\phi)^* A(\psi+\phi)\, d^3r = \int[A(\psi+\phi)]^*(\psi+\phi)\, d^3r$$

or $$\int\psi^* A\psi\, d^3r + \int\psi^* A\phi\, d^3r + \int\phi^* A\psi\, d^3r + \int\phi^* A\phi\, d^3r$$

$$= \int(A\psi)^*\psi\, d^3r + \int(A\phi)^*\psi\, d^3r + \int(A\psi)^*\phi\, d^3r + \int(A\phi)^*\phi\, d^3r$$

As A is Hermitian, hence

$$\int\psi^* A\psi\, d^3r = \int(A\psi)^*\psi\, d^3r$$

and $$\int\phi^* A\phi\, d^3r = \int(A\phi)^*\phi\, d^3r$$

$\therefore$ ~~$\int\psi^* A\phi\, d^3r + \int\phi^* A\phi\, d^3r =$~~

$$\therefore \int\psi^* A\psi\, d^3r + \int\phi^* A\phi\, d^3r = \int(A\psi)^*\psi\, d^3r + \int(A\phi)^*\phi\, d^3r$$

~~Again for the state function $(\psi+i\phi)$ we have~~

~~$$\int(\psi+i\phi)^* A(\psi+i\phi)\, d^3r = \int[A(\psi+i\phi)]^*(\psi+i\phi)\, d^3r$$~~

~~or $$\int\psi^* A\psi\, d^3r + \int\phi^* A\phi\, d^3r + i\int\psi^* A\phi\, d^3r - i\int\phi^* A\psi\, d^3r$$~~

~~$$= \int(A\psi)^*\psi\, d^3r + \int(A\phi)^*\phi\, d^3r + i\int(A\psi)^*\phi\, d^3r - i\int(A\phi)^*\psi\, d^3r$$~~

~~or $\int\psi^* A$~~

Now $$\cancel{\int(A\psi)^*\psi\, d^3r} + \int\psi^* A\phi\, d^3r + \int\phi^* A\psi\, d^3r + \cancel{\int(A\phi)^*\phi\, d^3r}$$

$$= \cancel{\int(A\psi)^*\psi\, d^3r} + \int(A\phi)^*\psi\, d^3r + \int(A\psi)^*\phi\, d^3r + \cancel{\int(A\phi)^*\phi\, d^3r}$$

$$\therefore \int\psi^* A\phi\, d^3r + \int\phi^* A\psi\, d^3r = \int(A\phi)^*\psi\, d^3r + \int(A\psi)^*\phi\, d^3r \qquad \text{(a)}$$

And if γ is also Hermitian, then

$$\int \beta^* \psi^* \cdot \gamma \phi \, dx = \int \gamma^* \beta^* \psi^* \cdot \phi \, dx \quad \text{--- (2)}$$

So that if β and γ are both Hermitian, then

$$\cancel{\int \psi^* \beta \gamma \phi \, dx} = \cancel{\int \beta^* \gamma^* \psi^*}$$

$$\int \psi^* \beta \gamma \phi \, dx = \int \gamma \beta^* \psi^* \cdot \phi \, dx \quad \cancel{\beta^* \gamma^* \psi^* \cdot \phi \, dx}$$

$$= \int (\gamma\beta)^* \psi^* \phi \, dx \quad \text{--- (3)}$$

$$= \cancel{\int (\beta\gamma)^* \psi^* \phi \, dx}$$

When β and γ Commute i.e — (4)

$$[\beta, \gamma] = 0$$

$$\beta\gamma - \gamma\beta = 0$$

$$\beta\gamma = \gamma\beta$$

So equation (3) Can be expressed as —

$$\int \psi^* \beta \gamma \phi \, dx = \int (\beta\gamma)^* \psi^* \cdot \phi \, dx \quad \text{--- (4)}$$

If A is a Hermitian operator, then we have

$$\int \psi^* A \phi \, dx = \int A^* \psi^* \phi \, dx \quad \text{--- (5)}$$

Comparing (4) and (5), we see that $\beta\gamma$ is Hermitian operator if β and γ commute.

~~Proof — We shall prove that if β and γ are commuting~~

Proof → ~~Let β and γ are two Commuting Hermitian operators.~~

~~We have to prove that~~

~~We shall prove that~~

We have to prove that if β and γ are Commuting Hermitian operators, then the operator $\beta\gamma$ is also Hermitian.

Let ψ and ϕ be two functions.

Consider the value of the integral $\int \psi^* \beta\gamma \phi \, dx$

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